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Using Average Historical Data for Risk Premium Estimates: Arithmetic Mean, Geometric Mean, or Something Else?

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In a variety of economic analyses, one often applies a discount rate to future cash flows for present value calculations. Under many scenarios, the discount rate used is the "required rate of return" calculated by the Capital Asset Pricing Model (CAPM). The CAPM states that this return/discount rate is a function of the risk free rate of return (RF), the company's volatility (β), and the market risk premium (MRP):

$$\text{Return} = RF + \beta\,(MRP) + \varepsilon$$

The risk free rate is typically the current rate on U.S. long term bond rates, and β is a measure of the company's volatility in comparison to the market as a whole.[1]

The market risk premium is intended to reflect the historical difference in returns from investing in the entire stock market and investing in long-term government bonds. The question of what measurement to use as the market risk premium has been the subject of past articles. Recently, Michael Julius looked at the differences between using the arithmetic mean or the geometric mean of the difference between long-term bond returns and stock market returns.[2]

Updating Julius' results through 1997, the difference between the arithmetic means of annual long-term bond returns and annual stock market returns is approximately 7.62 percent (e.g., 13.25 - 5.63 = 7.62). However, the market risk premium drops to 5.78 percent when the geometric means are compared (e.g., 11.00 - 5.22 = 5.78). Needless to say, this difference is not trivial, and could often change a valuation result by millions of dollars.[3]

Following standard statistical practice, these different means are calculated in the following manners:

1. The arithmetic mean is the traditional "average", where each of the 72 annual returns are added together and divided by 72.
2. The geometric mean is essentially the annual compounded return that would be needed to take the market from its original value in January 1926 to its ending value in December 1997 . That is, the geometric mean is a value x such that the Initial Market Value*$(1 + x)^{72}$ = Ending Market Value.

If all 72 annual returns were the same, then the arithmetic and geometric means would be equivalent. As the variance in historical annual returns grows, the difference between the arithmetic and geometric means increases. The most common way to measure variability is by standard deviation, and as the following table shows, larger standard deviations of the arithmetic means lead to larger differences between the arithmetic and geometric means.[4, 5]

Annual Returns: 1926-1997

Investment	Geometric Mean	Arithemetic Mean	Standard Deviation
Large Co. Stocks	11.00%	13.25%	22.14%
Small Co. Stocks[6]	12.71%	18.97%	46.01%
Long-Term Government Bonds	5.22%	5.63%	12.09%

Risk Premiums: 1926-1997

Investment	Geometric Mean	Arithmetic Mean
Large Stock Premium to Government Bonds	5.78%	7.62%
Small Stock Premium to Large Stocks	1.71%	5.72%

Using these data, risk premia are typically computed as the difference in mean returns (geometric or arithmetic) for stocks (large or small companies) and for long term government bonds. Therefore, both the large and small company market risk premia are very sensitive to the type of mean being used.

There are currently two main schools of thought in the use of means. Most valuation practitioners and academics focus on the arithmetic mean, but there is also a significant group that supports the use of the geometric mean.

Rather than choose between these two existing methods, Julius (1996) advocates a third procedure. His methodology is to consider the arithmetic means of annualized returns from rolling multi-year holding periods calculated from monthly data. By doing so, he finds rates that fall between the geometric mean and the arithmetic mean. He argues that using the three-year rate (which corresponded to the holding period for private stock under SEC Rule 144) and the 2-20 year combined rate (which reflects the uncertainty of timing the sale of private stock) lead to the most reasonable results. It is this search for a reasonable method other than strictly choosing between the arithmetic mean and geometric mean which most closely resembles the analysis below.

Approach 1: Arithmetic Mean

Ibbotson Associates, Shannon Pratt, and others espouse the arithmetic mean, stating that for the next year, the best guess for a return (of a stock or a bond) is its historical 1-year average. Therefore, this literature suggests that the arithmetic mean is the most appropriate estimate for next year's return. The same analysis applies two years into the future and so on. In this sense, each year in the future is treated independently with expected returns of the historical arithmetic mean of "R". Assuming the returns in each future year are independent, the expected return for investing in a stock for n years can be stated as $(1+R)^n$:[7, 8]

$$E[(1+R_1)(1+R_2)...(1+R_n)] = E[1+R_1]E[1+R_2]...E[1+R_n] = (1+R)(1+R)...(1+R) = (1+R)^n$$

That is, one would expect his initial wealth to be multiplied by $(1+R)^n$ in the next n years. This also implies a compounded annual rate (to be used for valuation purposes) of R, the historical arithmetic mean.

Approach 2: Geometric Mean

Damodaran[9] and others espouse the geometric mean since a discounted cash flow analysis is looking at the return over multiple years. Their point generally focuses on the limitations of using a one year measure (arithmetic mean) for discounting cash flows that are occurring two or more years into the future. Therefore, they suggest that one should be more concerned with initial and ending period wealth instead of each of the single returns. That is, investors are only interested in the expected compounded returns since that will estimate the wealth at the end of their investment.[10] The geometric mean calculates exactly that by determining the compound return that investors actually experienced over the past 72 years.

Alternate New Approach: Varies by Year

Like Julius, we suggest the expected rate of return or discount rate not be classified simply as either the arithmetic mean or the geometric mean. Rather, the rate can be tied to the number of years being discounted. The arithmetic mean divides historical data into 72 one-year periods, while the geometric mean divides the historical data into one 72-year period. Typically, in valuation or project finance, the relevant period of analysis is neither one nor 72 years, but somewhere in between. In fact, valuation projects usually look at cash flows for multiple years into the future.

The insistence upon using either the annual arithmetic mean or the 72-year geometric mean may be partially a result of the typical presentation of return data. That is, returns on stocks and bonds are typically presented with annual and total returns in the annual Ibbotson Yearbook. This makes it easy to use annual arithmetic means or 72-year geometric means, but not as simple to calculate other time periods or measures.

From a statistical/mathematical standpoint, the philosophy of arithmetic means is logical in that the best guess of the next observation is the arithmetic mean of a historical sample. That is, in trying to forecast a rate of return for next year, the best guess is the arithmetic mean of historical annual returns. In this sense, the historical data serve as an (unbiased) sample of the population of returns.

The term "return" is intentionally vague, however, as it does not define a time period. In attempting to forecast a return for one year into the future, a historical sample of one-year returns would provide an unbiased estimator. It would not be the same to take an average of historical 1/2-year returns and compound it twice. Nor would it be best to take an average of historical 1/4-year returns and compound it four times. Similarly, if one were attempting to forecast the returns for a two-year period into the future, the arithmetic mean of historical two-year returns would be a better estimator than historical one-year returns compounded twice. Because of business cycles, such differences can be significant. For example, for large company stocks from 1926-97, the (arithmetic) mean for five year returns was 64.5 percent, but the one-year (arithmetic) mean compounded for five years was 86.3 percent.

In summary, the arithmetic mean is generally the statistical "best guess" of return; however, this only holds if the returns being averaged to derive that arithmetic mean correspond (in years) with the number of years into the future for the cash flow in question.[11]

Implementation

Ibbotson Associates and others have published year-by-year returns for large company stocks, small company stocks, and government bonds for the past 72 years. To calculate the arithmetic mean returns for multiple year periods, one must first compound these annual return data to derive multi-year return data. For the purposes of comparison, such rates of return may also be annualized.[12]

Calculation

Ibbotson's data on small stock returns, large stock returns, and long-term government bond returns were considered for the calendar years 1926-97. Six different return time periods were considered: 1, 2, 3, 5, 10, and 72 years. For each period, rolling monthly returns were used. For example, the first 1-year period used was calendar year 1926. The next 1-year period used was end of January 1926 through end of January 1927. Each one-year period beginning each month was used until the last one-year period of calendar year 1997.

Annualized Market Risk Premiums: 1926-1997
(Arithmetic Means of Rolling Returns of Different Time Periods)

Cash Flow "n" years into the Future	Average Large Stock Premium over Government Bonds	Average Small Stock Premium over Large Stocks
1	7.62%	5.72%
2	6.57%	3.65%
3	5.97%	3.20%
5	5.38%	3.24%
10	6.01%	3.27%
72	5.78%	1.72%

For both large and small stocks, this table generally shows smaller market risk premiums as the number of years into the future increases. Exceptions to this are the 10- and full 72-year periods for large stocks, which have larger (annualized) risk premia than at 5 years. The 10-year period for small stocks is also larger than the 5-year period. This may be due to business cycles; however, further analysis is required. The later time periods (i.e., five and ten years) show significantly lower market risk premia than the simple (one-year) arithmetic mean, which will have highly positive effects on terminal value calculations.[13]

While the above table shows significant differences between large and small stocks over the 1926-1997 period, the returns are more similar across large and small stocks using more recent data from 1980-1997. In fact, using many time periods, small stocks actually had *lower* returns. The following chart measures market risk premiums from 1980-1997.

Annualized Market Risk Premiums: 1980-1997
(Arithmetic Means of Rolling Returns of Different Time Periods)

Cash Flow "n" years into the Future	Average Large Stock Premium over Government Bonds	Average Small Stock Premium over Large Stocks
1	4.82%	0.01%
2	3.58%	-1.08%
3	3.46%	-1.34%
5	2.85%	-3.16%
10	2.84%	-3.93%
18	5.37%	-1.01%

Conclusion

When choosing a discount rate to discount future cash flows for valuation purposes, there exist different situations in which one could use the geometric mean return or the 1-year arithmetic mean return. These two averages are measuring different time frames. Means have been calculated above which can correspond to most time periods used in discount cash flow analysis.

Endnotes

1. While the CAPM is commonly used in valuations as a baseline, adjustments are often made to the standard formula. Adjustments to CAPM include incorporating a specific company's risk factors and expected growth rates, as seen in Mercer, Z. Christopher, "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates: An Extension of Previous "Build-Up" Methodologies Based Upon The Capital Asset Pricing Model," *Business Valuation Review*, December 1989, Vol. 8, No. 4, pp. 147-156.
2. Julius, Michael, "Market Returns in Rolling Multi-Year Holding Periods An Alternative Interpretation of the Ibbotson Data," *Business Valuation Review*, June 1996, Vol. 15, No. 2, pp. 57-63.
3. These figures reflect the differences between large stocks and long-term government bonds using rolling 1 year returns calculated each month using data from 1926-1997 found in Ibbotson Associates, 1998 Yearbook, Chicago, Illinois, Appendix B.
4. There is no corresponding volatility measure for the geometric mean.
5. Computed using data for 1926-1997 from Ibbotson Associates, *1998 Yearbook*, Chicago, Illinois, Appendix B.
6. As defined by Ibbotson Associates, *1998 Yearbook*, Chicago, Illinois.
7. Pratt, Shannon P., Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 3rd. ed., Chicago: Irwin Professional Publishing.
8. Ibbotson provides an example using a discrete probability distribution for returns to make this point, *1998 Yearbook*, Chicago, Illinois, p. 154-155.
9. Damodaran, Aswath, "Damodaran on Valuation," 1994, John Wiley & Sons, Inc., New York.
10. Damodaran states, "In the context of valuation, where cash flows over a long time horizon are discounted back to the present, the geometric mean provides a better estimate of the risk premium." Damodaran, Ibid., p. 22.
11. For a cash flow that is 5 or 7 years into the future, objections might be raised since the number of data points for 5- or 7-year returns over the past 72 years is much less than that afforded by annual returns (e.g., 14 or 10 data points vs. 72). If there were such a concern about number of data points, practitioners would use the 144 semi-annual returns instead of 72 annual historical returns. In addition, once can overlap time periods by using both 1926-30 and 1927-31 as observations in the sample of 5-year returns, or use rolling monthly returns for even more data points.
12. The term "annualized" refers to the compounded annual return that would be necessary to earn the multi-year return. For example, if stocks have an average two year return of 30 percent, this can be annualized to 14 percent (i.e., $1.14^2 = 1.30$).
13. The issue of terminal value sensitivity is addressed in "Multiple Approached to a Valuation: The Use of Sensitivity Analysis," *Business Valuation Review*, Volume 15, No. 4, December 1996, pp. 157-160.

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