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Does a Small Firm Effect Exist when Using the CAPM? Not Since 1980 and Not when Using Geometric Means of Historical Returns

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Introduction

A significant topic of debate among finance and valuation professionals is whether or not a "small firm" effect exists. The debate hinges on whether small firms earn higher risk-adjusted returns than large firms, and as such, whether the required risk-adjusted rate of return for small firms is greater than that of large firms.

Typically, this issue arises in the context of applying the capital asset pricing model (CAPM) to estimate a company's (or project's) required rate of return or discount rate. There are many who believe the CAPM should not be used at all. Others feel the CAPM should only be used for large firms or should be adjusted to use with small firms since the explanatory power of the CAPM for historical returns has been shown to be lacking for small firms. However, through a variety of statistical testing procedures, there are others who now advocate the use of CAPM with no adjustment necessary for smaller firms.

This paper attempts to quantify when and under what assumptions a small firm effect exists. Using U.S. stock market data (primarily compiled by Ibbotson Associates), our results indicate that: (a) small firms earned significantly higher returns than large firms from 1926-79; (b) small firms have earned similar or lower returns than large firms since 1980; (c) small firms earned higher returns than large firms across the entire 1926-97 period;[1] and (d) small firms' higher returns over the 1926-97 period can be fully explained by the difference in their risk profiles (i.e., their betas) from those of large firms.

While the first three of the above findings can be derived from simple arithmetic using data compiled by Ibbotson Associates, the final result in some sense contradicts Ibbotson and many other sources. This discrepancy is due to how the CAPM is applied. This application involves, among other things, computing the historical difference between stock returns and government bond returns (i.e., the market risk premium over the risk free rate of return). If each of these returns is estimated using an arithmetic mean, the CAPM does not fully explain the returns of small stocks.[2] Our results indicate that use of a geometric mean for historical returns allows the CAPM to fully explain the historical returns of publicly traded stocks of all different sizes. This result is also significant in that it adds further critique to the exclusive use of arithmetic means when using CAPM.

From a practitioner's perspective, the results showing the better fit using a geometric mean could be significant in estimating rates of return/discount rates for smaller companies. Current convention is to (a) use the arithmetic mean in calculating a market risk premium and (b) apply a small firm premium to account for the fact that CAPM does not fully explain the risk adjusted returns for small firms. As detailed later in this paper, simply using the arithmetic mean calculates a higher market risk premium (and thus, higher rates of return/discount rates) than the use of a geometric mean. The addition of a small firm risk premium (which would not be necessary with the use of a geometric mean) would further distance the results of an "arithmetic mean" CAPM analysis from that of a "geometric mean" CAPM analysis. In fact, for small firms (with betas of 1), this difference can be up to seven percentage points (five percentage points for small firms, two percentage points for differences across means).

This paper is divided into five sections. This first section provides an overview and executive summary of the paper. A literature review of several papers advocating and critiquing the small firm effect is presented in the second section. The CAPM, and the source data for its various elements is described in the third section. The fourth section applies the Ibbotson data to test the small firm effect over different time periods and with respect to different calculations of historical means (i.e., arithmetic vs. geometric.) Concluding remarks with examples are provided in the fifth section.

Literature Review of Small Firm Effect

Across different stock exchanges and time periods, some researchers have found that small firms have higher historical returns than large firms, while other researchers have found no such evidence. Some of the later type are summarized below.

Koutoulas and Kryzanowski, in a 1996 article for *Financial-Review*, studied size-ranked portfolios of all shares traded on the Toronto Stock Exchange between March 1962 and March 1988. Firms trading on the Toronto exchange were adjusted by allowing the risk premia to vary in proportion to the conditional volatilities of the macro-economic innovations that follow an autoregessive specification. The study found that the small-firm effect is absent in risk-adjusted returns.

In a 1993 study for the Federal Reserve Bank of Kansas City, Shen concluded that the small-firm effect disappears entirely when the bid-ask spread (part of the transaction cost and the measure of the liquidity of financial assets) is included in regressions of equilibrium asset returns. The study found that once the effect of transaction costs is properly accounted for, firm size becomes irrelevant.

Another relevant study, completed by Ghosh for the Fall 1992 edition of the *Journal of Financial Research*, demonstrated that, using daily stock rate of return data for 1985 through 1989 (excluding October 1987 and October 1989), estimates of beta coefficients are overstated significantly and could explain, in part, the small firm effect.

Fortune wrote an article for the March-April 1991 issue of the *New England Economic Review*, in which he explains that any anomalies of the efficient market hypothesis (such as the small firm hypothesis) can be explained by resorting to the model of "noise trading." In the noise trading model, markets are segmented with best informed traders enforcing efficiency in the pricing of large firm stocks while less informed traders dominate the market for small firms. According to the study, the model can generate cycles in stock prices similar to those observed in reality.

The CAPM and its Data Sources

The CAPM describes the relationship between the returns in an individual stock and the market as a whole. The CAPM essentially creates a "security market line" which provides a linear relationship between a stock's required rate of return and its risk (i.e., its beta). Beta is a measure of a firm's volatility relative to the market as a whole.

The CAPM may be stated as:

$$\text{Required Rate of Return} = RF + \beta\,(MRP)$$

Where RF = the risk-free rate, β = the asset's beta and MRP = the market risk premium, or the historical difference between the return to the market as a whole and the risk-free return.

In applying the CAPM formula, practitioners make use of publicly available current and historical information. A variety of sources calculate companies' betas, typically based on the previous 5 years of monthly stock prices (i.e. 60 months of data). The risk free rate is the prevailing rate on long term government bonds. The market risk premium is measured as the historical difference between investing in the stock market and investing in long-term government bonds. As described in the section below, the most commonly used source for such historical information is Ibbotson Associates' Annual Yearbook.

Data Analysis

While the CAPM is likely the most commonly used method to determine cost of capital, it is often criticized for not incorporating certain features of the market and/or not fully explaining the risk of certain types of companies. Among valuation practitioners, perhaps the most important issue is whether the CAPM incorporates the risk associated with smaller firms.

Ibbotson has shown that smaller firms have historically earned higher market returns than larger firms. Specifically, from 1926-97, small firms outperformed large firms by 4.7 and 1.7 percentage points when averaging arithmetically or geometrically, respectively.[3] As such, it is common to see these percentages added to the CAPM result in the valuation of smaller companies.

This direct comparison, however, does not tell the entire story, because small stocks tend to have higher betas than larger stocks. Therefore, given higher betas for small firms, the CAPM alone would predict that the small firms would have higher returns than large stocks. Using arithmetic means, however, the CAPM still underpredicts the returns of smaller firms. Ibbotson, in fact, devotes an entire chapter to this point and shows that for the smallest of firms, the CAPM underestimates returns (see Table 1). Under this analysis, "micro-cap" (defined as the lowest 20 percent of market capitalization) firms would require an additional 3.3 percent size premium and firms in the lowest 10 percent would require an additional 5.4 percent premium. While such results could be directly added to a CAPM analysis, it should be cautioned that Ibbotson only performs this analysis using the arithmetic means of historical returns.

The issue of using arithmetic or geometric means has been discussed in several previous works. Most, including Ibbotson, advocate using the simple arithmetic mean of one-year returns, as this represents the expected value of the annual return of the asset one year into the future. However, although the arithmetic mean of annual stock returns can be interpreted as the expected return on stocks over the next 12-month period, it cannot be converted into a compound annual rate of return over periods longer than one year.[4]

Becker and Gray (1998) recommend using the arithmetic mean of rolling n-year periods to estimate the expected value of annual returns n years into the future. In the case of summarizing 72 years of returns (as used by Ibbotson in quantifying a small firm risk premium), or in computing a terminal value, which by its very nature is a long-term concept, that article would suggest using a geometric mean approach. This paper underscores the possibility that compounding an arithmetic mean of annual returns will not be representative of multi-year returns due to the effects of percentages.[5]

It is interesting to note that the small firm premia quantified by Ibbotson do not exist when the historical data being applied are averaged in a geometric manner. As seen in Table 2, the CAPM is always within 0.7 percentage points of predicting the historical returns of firms in each of the 10 deciles. This result may be even more visually apparent when comparing the "tight fit" of the security market line in Figure 2 (geometric mean) with that of Figure 1 (arithmetic mean.)

While such results suggest that the CAPM can fully explain small firm returns over the 1926-97 period when using geometric means, it does not necessarily imply that small firms and large firms should be compared like apples and apples. That is, for the 1926-97 period small firms, even using geometric means, had higher betas and higher returns than large firms. If the beta difference could be fully explained by higher debt levels, this would allow an unadjusted small/firm large firm comparison because betas are "relevered" based upon the company's amount of debt. During this time period of 1926-97, however, beta differences could not fully be explained by debt/equity differences.[6] This suggests that during this time period: (a) CAPM does fully explain returns from all sized companies, but (b) smaller companies inherently had higher betas than larger companies. A small firm adjustment using these data might adjust the betas of some of the comparables or target company, rather than by adding a small company premium directly to the CAPM result.[7]

While the above analysis shows that the CAPM can fully explain the returns to small firms (as well as large firms) when using the geometric mean, it is also worth examining the difference in small and large firm returns in some detail.[8] While small firms outperformed large firms during the entire 72-year period of 1926-97, Table 3 shows that large firms actually outperformed small firms in the 1980s and 1990s (through December 31, 1997). In fact, the 10th decile of firms (the smallest 10 percent of the market) earned nearly 6 percentage points less than the market as a whole during this time. If one were to focus the analysis of historical stock performance on this information, it is unlikely that any type of small firm adjustment (beta or otherwise) could generally be considered appropriate.

Conclusions

Using U.S. stock market data (primarily compiled by Ibbotson Associates), our results indicate that: (a) small firms earned significantly higher returns than large firms from 1926-79; (b) small firms earned similar or lower returns than large firms since 1980; (c) small firms earned lower returns than large firms across the entire 1926-97 period; and (d) small firms' lower returns over the 1926-97 period can be fully explained by the difference in their risk profiles (i.e., their betas) from those of large firms. Thus, only under relative limiting assumptions and time periods can the existence of a small firm effect be shown.

Ibbotson's Annual Yearbook includes a graph that compares the accuracy of the CAPM by decile. This is a graphic demonstration of an argument for the small firm premium and it is shown in Figure 1. It is clear that using the CAPM with arithmetic means is not sufficient. It is this fact that many practitioners use the CAPM and then add an additional premium to the CAPM to adjust for firm size. It is clear by this graph that the smaller the firm, the larger the discrepancy between the CAPM prediction and the known historical returns.

What happens if we repeat the above exercise using geometric means instead of arithmetic means? Figure 2 is the CAPM prediction line along with the actual historical geometric means. It is clear that CAPM is a much better predictor of the historical geometric return than the arithmetic return. Under the geometric mean analysis, there is no need to make any adjustment to the CAPM predicted return based on firm size. The difference in betas fully explains the difference in returns across size.[10]

From the practitioner's perspective, this finding should provide further evidence that using the arithmetic mean with a small firm premium added to the CAPM as the only measure of required return is perhaps worth reexamination. Further research into this area (especially with regard to very small "mom and pop" firms) is warranted.

Biography

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Endnotes

1. This particular result was already stated and calculated by Ibbotson Associates, and required no analysis on our part.
2. As detailed later, the CAPM's predicted return is approximately 5 perentage points below the returns actually earned by the smallest decile of publicly traded stocks across the 1926-97 period.
3. The "smaller" companies are defined as the smallest 30 percent in terms of market capitalization on the New York Stock Exchange. Ibbotson Associates, "Stocks, Bonds, Bills, and Inflation 1998 Yearbook," 1998, Ibbotson Associates.
4. Siegel, Jeremy, "The Equity Premium: Stock and Bond Returns Since 1802," *Financial Analysts Journal*, January-February 1992, pp. 28-46.
5. An example of this can be seen for an initial investment of $100. Suppose this investment lasted for 10 years, with 5 years of 50% returns and 5 years of -50% returns. While a 0% return would be statistically accurate for this time period, it would be deceiving in that the investment would have decreased in value from $100 to $23.73! More representative would be the geometric mean of this 10 year return, which would be -13.4%.
6. Roger Grabowski and David King, in their article "New Evidence on Size Effects and Rates of Return", show that debt levels are quite similar across company size using a variety of measures, but betas are higher for smaller firms.
7. For example, suppose that the target company is in the 10th decile and a comparable company is in the 5th decile of size. Based on the Grabowski, et al. article above, it is shown that betas of companies in the 10th decile are 20 percent higher than those in the 5th decile (after adjusting for debt.) As such, the comparable's levered beta of 1.2 should be adjusted to 1.44 before unlevering and relevering to the target company's capital structure.
8. While such differences alone should not require a small firm premium (due to differences in β's), this historical difference is often applied nonetheless.

9. This comment corresponds to returns being measured in a geometric manner, but Table 3 shows that the results are similar when using arithmetic returns.

10. As stated above, however, there may still be some rationale for not simply comparing small and large firm returns on an "apples to apples" basis.

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Tables and Figures

Table 1 – CAPM Results using Arithmetic Mean (1927-1997)

Decile	Beta	CAPM Predicted Return	Actual Return	Difference
1 (Largest)	0.9	12.5	11.9	-0.6
2	1.04	13.6	13.7	0.1
3	1.09	13.9	14.3	0.4
4	1.13	14.2	15.0	0.8
5	1.16	14.8	15.8	1.0
6	1.19	14.7	15.8	1.1
7	1.24	15.1	16.4	1.3
8	1.28	15.4	17.5	2.1
9	1.35	15.9	18.2	2.3
10 (Smallest)	1.46	16.8	21.8	5.0

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Table 2 – CAPM Results using Geometric Mean (1927-1997)

Decile	Beta	CAPM Predicted Return	Actual Return	Difference
1 (Largest)	0.9	10.2	10.2	0.0
2	1.04	10.9	11.3	0.4
3	1.09	11.2	11.7	0.5
4	1.13	11.4	11.9	0.5
5	1.16	11.6	12.3	0.7
6	1.19	11.7	12.1	0.4
7	1.24	12.0	12.2	0.2
8	1.28	12.2	12.4	0.2
9	1.35	12.6	12.5	-0.1
10 (Smallest)	1.46	13.2	13.9	0.7

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Table 3 – Comparison of Results Since 1980

	1926-1997		1980-1997	
	Arithmetic	Geometric	Arithmetic	Geometric
Large Stocks	13.0	11.0	17.6	16.3
Small Stocks	17.7	12.7	17.6	14.9

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Figure 1 – CAPM Prediction Using Arithmetic Mean

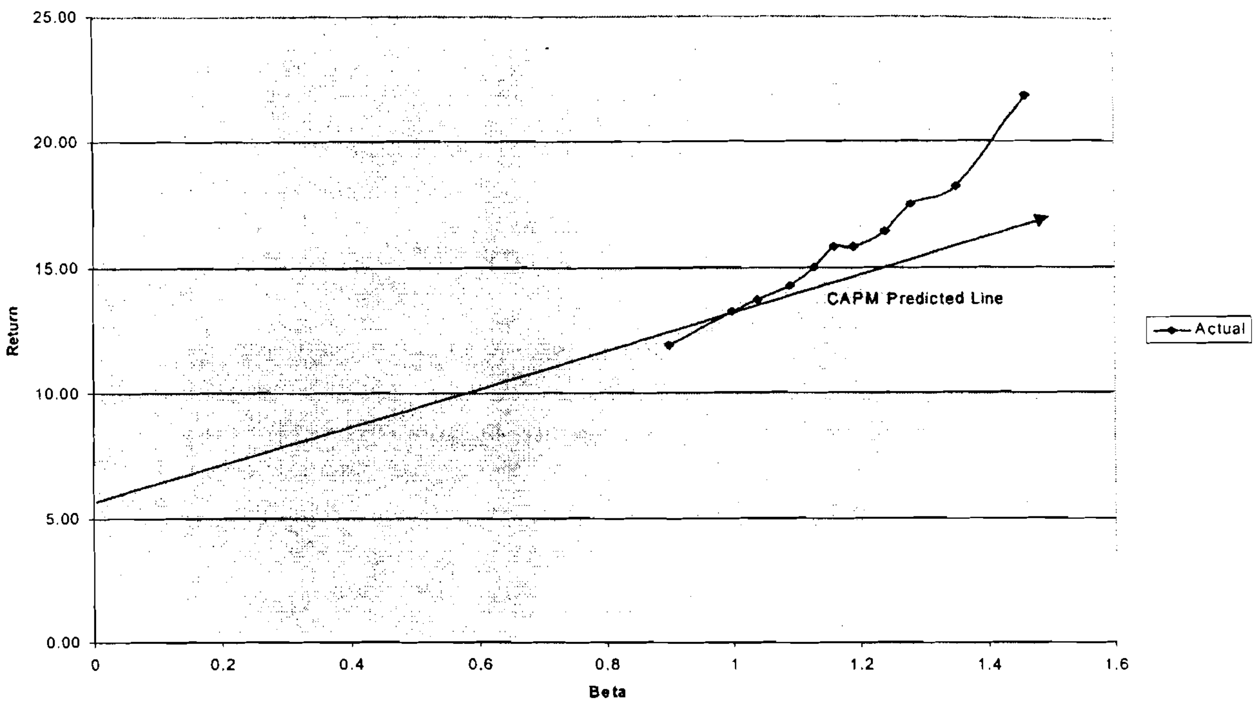

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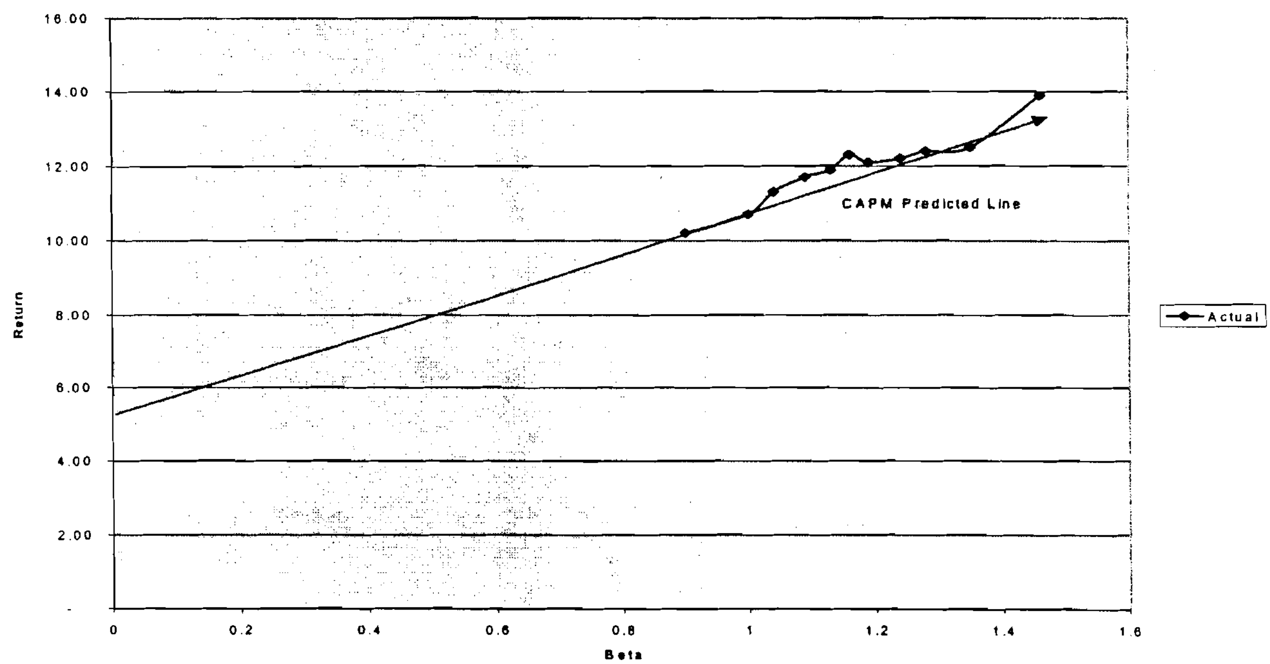
Figure 2 – CAPM Prediction Using Geometric Mean



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